Exploiting the joint distribution of amplitude and monopulse ratio for chi-square fluctuating targets for target DOA estimation

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***Abstract*— This paper deals with the Direction of Arrival (DOA) estimation for chi-square fluctuating targets. The joint Probability Density Function (PDF) of the amplitude in the sum channel and the monopulse ratio for a chi-square fluctuating targets presented in [1] is here used to derive several DOA estimators. The proposed estimators are obtained by following respectively the Maximum Likelihood criterion, the Method of Moments, and the Maximum Entropy Method. Their performance are characterized in terms of bias and standard deviation error.**

I. Introduction

This paper deals with the Direction of Arrival (DOA) estimation for chi-square fluctuating targets. In [1] the joint Probability Density Function (PDF) of the amplitude in the sum channel and the monopulse ratio for a generalized Swerling target model have been derived. Moreover, they have been exploited to derive optimum track initiation schemes. In this paper the PDF of the real part of the monopulse ratio (η) conditioned to the received amplitude on the sum channel (t) is used to derive several estimators for the target DOA based on the Maximum Likelihood criterion, the Method of Moments, and the Maximum Entropy Method. The derived PDF is then used to characterize the estimators performance in terms of bias and standard deviation error.

The paper is organized as follows. In Section II the joint PDF of the amplitude in the sum channel and the monopulse ratio for a generalized Swerling target model is recalled together with its marginal distributions. Section III is devoted to derive and characterize the DOA estimators, while in Section IV their performance are assessed and compared. Finally in Section V we draw our conclusions.

II. Joint distribution of monopulse ratio and amplitude

Using a monopulse antenna in a radar system, two complex numbers are available for each range gate, corresponding to the sum and difference channels (assuming only one difference beam is used). In [1] the joint PDF of the amplitude in the sum channel and the monopulse ratio for a generalized Swerling target model (i.e. considering the generic Chi-Square [2] fluctuation model for the target Radar Cross Section) has been derived. Specifically, for a target with $\delta=\Delta(\theta)/\Sigma(\theta)$, where $\Delta(\theta)$ and $\Sigma(\theta)$ are the delta and sum beam pattern respectively, the joint pdf of the real part of the monopulse ratio η and the received amplitude t is:

$$p(t,\eta \mid S_o,\delta;H_1)=\frac{2}{\sqrt{\pi}\,(1+S_o)^m}e^{-\frac{(\eta-\delta)^2+(1+\eta^2)/S_o}{1+1/S_o+\delta^2}t^2}\cdot \\ \cdot\sum_{l=0}^{m-1}\frac{(1+\eta\,\delta)^{2l}}{(1+1/S_o+\delta^2)^l}\cdot t^{2(l+1)}\cdot C_l(S_o,\delta;m) \quad (1)$$

where the terms $C_l(S_o,\delta;m)$ only depend on S_o, δ and m:

$$C_l(S_o,\delta;m)=\frac{1}{2^{2(m-1-l)}(2l)!}\cdot \\ \cdot\sum_{i=l}^{m-1}\frac{[2(m-1-i)]!\;(2i)!}{i!\;[(m-1-i)!]^2(i-l)!}\left(\frac{1+1/S_o}{1+1/S_o+\delta^2}\right)^{i+\frac{1}{2}} \quad (2)$$

being m a parameter that defines the fluctuation level of the Swerling model, and $S_0=SNR/m$.

The marginal distributions can be obtained by integrating the joint PDF with respect to the other variable. Using some combinatorial transformations, the multiple summations can be removed yielding for the PDF of the instantaneous received amplitude t, [1]:

$$p(t\mid S_o;H_1)=\int_{-\infty}^{+\infty}p(t,\eta)\,d\eta= \\ =2e^{-\frac{t^2}{1+S_o}}\sum_{n=0}^{m-1}\binom{m-1}{n}\frac{1}{n!}\frac{S_o^{\,n}}{(1+S_o)^{m+n}}\,t^{2n+1} \quad (3)$$

that is well known for the instantaneous received power (i.e. t^2) of Swerling I, II, III, and IV targets (e.g. [3]-[4]). Similarly, for the marginal PDF of the measured monopulse ratio η:

$$p(\eta\mid S_o,\delta;H_1)=\int_0^{\infty}p(t,\eta)\,dt=\frac{(1+1/S_o+\delta^2)^{\frac{3}{2}}}{(1+S_o)^m}\cdot \quad (4)$$

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