Fractal Based Detection Using Blind Box-Counting Method in High Resolution Radars

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Abstract— **Sea clutter refers to the radar returns from the sea surface. Accurate modelling of sea clutter and detection of low observable targets within sea clutter are major problems in remote sensing and radar signal processing applications. Recently fractal geometry is applied to the analysis of high range resolution radar sea clutters. The box-counting method is widely used to estimate fractal dimension but it has some drawbacks rarely considered in literature. We explain the problem of box size range and present a novel method to select an appropriate range.**

I. Introduction

Due to massive reflection of radar pulses from wavy ocean surfaces, sea clutter is often highly non-Gaussian [1–8], even spiky [9], especially in heavy sea conditions. Traditionally, sea clutter is often studied in terms of certain simple statistical features, such as the marginal probability density function (pdf). The non-Gaussian feature of sea clutter has motivated researchers to employ Weibull [1], log-normal [2–4], K [5–7, 10], and compound-Gaussian [8] distributions to model sea clutter. However, such simple phenomenological modelling of sea clutter only offers limited analytical or physical understanding.

To gain deeper understanding of the nature of sea clutter, the concept of fractal has been employed for the modelling of the roughness of sea surface and investigation of scattering from rough surface [11–13].

Since the ultimate goal of sea clutter study is to improve detection of targets embedded within clutters, a lot of effort has been made to design innovative methods for target detection within sea clutter. Notable methods include time-frequency analysis techniques [14], wavelet based approaches [15], neural network based approaches [16–18], and wavelet-neural net combined approaches [19], as well as utilizing the concept of fractal dimension [20] and boxing-counting based multi-fractal analysis [21].

In this paper we examine the accuracy of estimated fractal dimension using box counting approach and emphasize the effect of Box Size Range (BSR) on it. In order to compare the estimated and actual fractal dimension of a signal, we employ band-limited Weierstrass-Mandelbrot (WM) functions in simulations. They are the basic functions in sea clutter model of Berizzi [12]. Due to finite resolution, investigations confirm that a fractal signal (e.g. sea clutter data) exhibit fractal behaviors in a specific BSR in box-counting method. Simulations show that for different WM functions (different fractal dimension), there are distinct appropriate BSRs. In detection theory context, to detect a target in clutter two different BSRs should be employed in fractal dimension estimation for the two hypotheses (pure clutter and noise versus target in clutter and noise). To the best knowledge, all of the efforts that has been done in utilizing box-counting method for fractal dimension estimation [11, 19, 20, 22, 23] are based on a heuristically pre-defined BSR for all scenarios.

At the end, we propose a novel approach so called Blind Box-Counting method which adaptively calculates fractal dimension of a signal without fixing a BSR. Performance comparison of the new detector and a traditional one [23] proves significant improvement.

The rest of the paper is organized as follows. In Sec. II, we briefly describe the fractal based detection using box counting method. In Sec. III, we analyze box size range effect on dimension estimation. In Sec. IV, we introduce blind box counting method. Finally, some concluding remarks are made in Sec. V.

II. Fractal Based Detection Using Box Counting Method

As we know, there are several kinds of fractal dimensions like Hausdorff, Box-Counting, Entropy and Correlation dimensions. Fractal dimension based on the Box-Counting is the most feasible and easiest way of dimension calculation. Applying the fractal dimension method in signal detections, we can easily set up a detector which compares the estimated Fractal dimension with a threshold and accepts or rejects the hypothesis of existence of the target in the received signal.

A. Box Counting Dimension

In fractal geometry, the Minkowski–Bouligand dimension [24], also known as Minkowski dimension or box-counting dimension, is a way of determining the fractal dimension of a set S in a Euclidean space R^n, or more generally in a metric space (X, d).

Imagine this fractal lying on an evenly-spaced grid (Fig. 1), and count the number of boxes required to cover the set. Suppose that Nr(ε) is the number of boxes of side length ε required to cover the set. Then the box-counting dimension is defined as:

$$\dim_{Box} f = D = \lim_{\varepsilon \to 0} \frac{\log(Nr(\varepsilon))}{\log(1/\varepsilon)} \tag{1}$$