New Generation Doppler Radar Processing : Ultra-fast Robust Doppler Spectrum Barycentre Computation Scheme in Poincaré's Unit Disk

Christrian Chaire[#], Frédéric Barbaresco[*]

[#]*Thales Airborne Systems*
10 avenue de La 1ere D.f.l., BP 58324, F-29200 Brest, France
christian.chaure@fr.thalesgroup.com
[*]*Thales Air Systems*
Hameau de Roussigny, F-91470 Limours, France
frederic.barbaresco@thalesgroup.com

Abstract— **In case of short time series, we prove that we can replace classical Radar Doppler processing chain based on FFT by a new generation processing based on Lie Group Theory. We have defined a robust and ultra-fast computation of Doppler Spectrum Barycentre based on "Median Spectrum" estimation through an regularized autoregressive identification in lattice and ladder form like BURG algorithm.**

I. Introduction

In Doppler radar analysis, the received signal consists of complex samples formed of M pulses coming from M reflections on the targets of the incident radar beam during the antenna scanning. When M is very low, the classical FFT processing is ineffective for spectral analysis and for extracting targets out of the clutter. In such a situation a complex AR identification is very helpful and is used since a long time. An effective separation of the targets from the noise in an homogeneous area presuppose identifying a central element in a sense which must be defined, representing the clutter. After this calculation, an appropriate distance and a threshold may separate the targets and clutter, (TFAC).

In this purpose, the Geometric Information Theory on riemannian manifolds is involved to define the median of P objects in an abstract set which is a differentiable manifold connected to the result of complex AR in lattice form. Then, an iterative gradient algorithm can process for getting a convergent element representing the median.

The classical processing takes for model the radar samples of M pulses as M-dimensional complex gaussian vectors with a zero-mean value and a M-dimensional hermitian correlation matrices in Toeplitz form. And Geometric Information Theory is involved to get a median element inside a set of hermitian matrices. But the resulting algorithms are very heavy in means of calculation. Here this problem is by-passed thanks to three craftiness:

First craftiness: To transform the empirical correlation matrices if they exist, in suites of complex coefficients of reflection of an AR identification in lattice form which are numbers in the Poincaré disk D. This disk consists of z complex with |z|<1. These coefficients may be also the starting of the calculation, if no correlation matrix is available.

Second craftiness: To use a specific riemannian metric defined as the opposite of the hessian of the entropy of a M dimensional complex gaussian distribution in relation to the coefficients of reflection. This stage differs from the classical Geometric Information Theory based on Fisher Information Matrix used as the metric tensor [2][3][4][5].

Third craftiness: To use the Möbius transform operating in the Poincaré disk D which sends general geodesics in D onto segments of radius in D. Thanks to that the calculation of the iterative gradient algorithm is strongly facilitated .

The resulting algorithm is very simple, without iterative matrix computation, while being of a big efficiency..

II. Median and Barycentre on a Riemannian Manifold

M points, called anchorage points, being given on a riemannian manifold, the average Mb is defined as a point minimizing the sum of the squares of the geodesic distances between Mb and the anchorage points [1]. A median point Me minimizes the sum of the geodesic distances between the point Me and the anchorage points. If d(ai,x) is the geodesic distance between a anchorage point "ai" and the point "x", then d(x, ai) is derivable with regard to x except at the point " ai ". Its gradient vector is a unitary vector in the local metric meaning, tangent in the manifold in x and tangent in the geodesics steered of "ai" toward "x".

The average (resp median) is reached by a gradient algorithm cancelling the gradient of $J_{mo}(x)$ (resp $J_{me}(x)$) :

$$J_{mo}(x)=\frac{1}{M}\sum_{i=1}^{M}d(a_i,x)^2 \quad \text{and} \quad \nabla J_{mo}(x)=\frac{2}{M}.\sum_{i=1}^{M}d(a_i,x).\vec{u}_i(x)$$

$$J_{me}(x)=\frac{1}{M}\sum_{i=1}^{M}d(a_i,x) \quad \text{and} \quad \nabla J_{me}(x)=\frac{1}{M}\sum_{i=1}^{M}\vec{u}_i(x)$$

In R^n the gradient algorithm would spell: $x_{j+1}=x_j-\gamma_j.\nabla J(x_j)$

On a riemannian manifold the algorithm spells: $x_{j+1}=\exp_{x_j}(-\gamma_j.\nabla J(x_j))$ Where "$\exp_x(U)$" is the riemannian exponential chart which is the movement along the geodesic coming from x headed toward the tangent vector U in x, on a distance equal to the modulus of U..