Quasi-Analytical Method For Estimating Low False Alarm Rate

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***Abstract*— This paper proposes a new approach for estimating low false alarm rate (FAR) using Quasi-Analytical (QA) method. The results from both theoretical analysis and simulation show that QA estimation is unbiased with obvious simulation speed improvement with respect to Monte Carlo (MC) and Conventional Importance Sampling (IS) for typical application cases. Furthermore, unlike IS approaches that rely on complex optimization procedures for optimal IS parameters or sub optimal parameters, the QA is simple to implement and computationally effective.**

I. Introduction

In any radar system with automatic detection, the FAR needs to be controlled in very low probability range. The MC method is commonly used in the simulation for FAR. In order to estimate very low error rate, a huge number of MC samples are inevitably needed. Hence, various IS estimation techniques [1-3] have been proposed for estimating low FAR.

In any low probability test, MC simulation is very slow because only a few simulation samples contribute to error events in the statistics. For example, if we use 1E6 simulation samples to estimate a false alarm of 1E-6, in average, only one simulation sample contributes to the error event for the FAR estimation. To speed up the simulation using IS, the random inputs are modified to force the error events happen more often. Then a weighting function is used to keep the estimation unbiased. As a result, the IS simulation is more efficient than MC. In the proposed QA estimation all simulation samples contribute to the FAR statistics and the QA method is therefore very efficient.

In [4], the QA method was introduced for communication system and signal integrity test. In this paper, we will use the QA method for radar FAR estimation. A QA estimator will be constructed and its statistical properties will be analysed to make sure that it fits for low false alarm estimation. Application examples will show the advantage of the QA method in comparison with MC and IS approaches.

II. Estimators Based on MC and IS Approaches

Consider a radar constant false alarm rate (CFAR) detector. Let $x_0, x_1, ..., x_M$ denote the random inputs consisting clutters and noise, where M+1 is the total number of random inputs. For the detection threshold T, the false alarm test can be described as

$$\frac{x_0}{g(x_1, x_2, ..., x_M)} \geq T \tag{1}$$

Assume that the analysis cell under test and each auxiliary sample x_i for *i=1, M* are independent identically distributed. The function of g can be defined in different ways for different types of CFAR systems. Consider a cell averaging (CA) CFAR detector. In this case, the function g is a simple linear summation and (1) can be re-expressed as

$$\frac{x_0}{z} \geq T / M \tag{2}$$

where $z = \sum_{i=1}^{M} x_i$ (2a)

The FAR is given by

$$P_{fa} = \iint\limits_{x_0/z \geq T/M} f_1(x_0) f_2(z) dxdz \tag{3}$$

where $f_1(.)$ is the probability density function (pfd) of x_i for *i= 0,.., M* and $f_2(.)$ is the pdf of z. For the FAR estimation, different estimators can be constructed as below.

A. MC Estimator

Based on (3), we can construct a MC estimator as

$$\hat{P}_{fa\_mc} = \frac{1}{N} \sum_{j=1}^{N} D(x_0 / z - T / M) \tag{4}$$

where *N is* the number of simulation samples in the MC simulation and the indicator function *D(t)* is used, which equals 1 if *t* >0, but otherwise equals 0. It is simple to show that the MC estimator is unbiased and the MC simulation variance is given by

$$\sigma_{MC}^2 = P_{fa}(1 - P_{fa}) / N \tag{5}$$

B. Conventional IS estimator

To speed up the FAR simulation, the pdf of the input random variables can be modified, so that the false alarm events occur more frequently. Thus, the number of simulation samples can be reduced. Of course, the count of false alarm events must be properly weighted to obtain an unbiased estimation of the FAP. The simplest way [1] to modify the pdf is to increase the variance of inputs, denoted as conventional