Reply on Comments of Reviewer 1

Comment 1

1) reference 4 still has not been modified. The reference given in the paper is inadequate and will be difficult to obtain by readers. The full ref I gave in the first review should be used.

The authors agree with this comment and replace the reference:

[4] Lueneburg E. "Radar Polarimetry: A Revision of Basic Concepts". Int. Workshop on Direct and Inverse Electromagnetic Scattering, Marmara Research Center, Gebze-Turkey, 1995

by

[4] Serbest, A.H., Cloude S.R. (eds). "Pitman Research Notes in Mathematics", Vol. 361, Longman 1996, pp. 257-275

Comment 2

2) The authors have not related their main results (equations 16 and 25) to the pauli matrices. These matrices are widely used by the community to analyse polarimetric problems and so to make a connection between your work and the algebra of these matrices would help clarify your contribution.

In order to answer this remark, the authors rewrite this part:

I. DELETE the paper text selection (starting from the first paragraph before Eqn. (10) up to Eqn. (13) inclusive

~~From elementary matrix theory it is known that any asymmetric matrix can be decomposed into a symmetric ($\mathbf{S}^{(s)}$) and a skew-symmetric ($\mathbf{S}^{(a)}$) component. The initial scattering matrix (9) is written as:~~

$$\mathbf{S} = \mathbf{S}^{(s)} + \mathbf{S}^{(a)}, \tag{10}$$

~~where~~

$$\mathbf{S}^{(s)} = 0.5 \cdot (\mathbf{S} + \mathbf{S}^{T}), \tag{10a}$$

$$\mathbf{S}^{(a)} = 0.5 \cdot (\mathbf{S} - \mathbf{S}^{T}). \tag{10b}$$

~~It is easily seen that these matrices take the form~~

$$\mathbf{S}^{(s)} = \begin{bmatrix} \dot{S}_{11} & 0.5 \cdot \left(\dot{S}_{12} + \dot{S}_{21}\right) \\ 0.5 \cdot \left(\dot{S}_{12} + \dot{S}_{21}\right) & \dot{S}_{22} \end{bmatrix}, \tag{11}$$

~~and~~

$$\mathbf{S}^{(a)} = \dot{\Delta} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \tag{12}$$

~~where Δ is the complex weighting coefficient~~

$$\dot{\Delta} = 0.5 \cdot \left(\dot{S}_{21} - \dot{S}_{12}\right). \tag{13}$$

II. INSERT the following text instead of the deleted selection

We now decompose **S** by using the orthogonal system of Pauli matrices

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\ \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ \sigma_3 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}, \tag{10}$$

so that the matrix (9) takes the form

$$\mathbf{S} = A_0\,\sigma_0 + A_1\,\sigma_1 + A_2\,\sigma_2 + A_3\,\sigma_3 = \sum_{i=0}^{3} A_i\,\sigma_i\ , \tag{11}$$

where

$$A_i = 0.5\cdot \mathrm{Sp}\left[\mathbf{S}\,\sigma_i\right]. \tag{12}$$

Thus, the scattering matrix **S** can be represented

$$\mathbf{S} = 0.5\left\{\left(\dot S_{11} + \dot S_{22}\right)\sigma_0 + \left(\dot S_{11} - \dot S_{22}\right)\sigma_1 + \left(\dot S_{12} + \dot S_{21}\right)\sigma_2 + \left(\dot S_{21} - \dot S_{12}\right)\sigma_3\right\}. \tag{13}$$

The first three terms of the decomposition (13) describe the symmetric component $\mathbf{S}^{(s)}$

$$\mathbf{S}^{(s)} = \begin{bmatrix} \dot S_{11} & 0.5\cdot\left(\dot S_{12} + \dot S_{21}\right) \\ 0.5\cdot\left(\dot S_{12} + \dot S_{21}\right) & \dot S_{22} \end{bmatrix}, \tag{14}$$

and the fourth term is the skew-symmetric component

$$\mathbf{S}^{(a)} = \begin{bmatrix} 0 & -j\,0.5\cdot\left(\dot S_{21} - \dot S_{12}\right) \\ j\,0.5\cdot\left(\dot S_{21} - \dot S_{12}\right) & 0 \end{bmatrix} = j\,\dot\Delta\cdot\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \tag{15}$$

where

$$\dot\Delta = 0.5\cdot\left(\dot S_{21} - \dot S_{12}\right). \tag{16}$$

As a result, the origin of matrix is written as

$$\mathbf{S} = \mathbf{S}^{(s)} + \mathbf{S}^{(a)}. \tag{17}$$

Since the "j" factor in (15) is not important in our further analysis, it will be omitted hereinafter.

III. RENUMBER all subsequent equations and the corresponding cross-references in the paper (the renumbering is marked out by green color in the corrected version)

IV. INSERT the text in the paragraph that was after Eqn. (18) in the previous version (Eqn. (22) in the corrected version with the renumbered equations), so that this paragraph takes the form:

Since the ellipticity and orientation angles are chosen arbitrarily, it is possible to conclude that the second item in (22) with the proportional factor $\dot\Delta = 0.5\cdot(\dot S_{21} - \dot S_{12})$ will not depend on ε', θ' parameters in the transformation matrix $\mathbf{U}'$. Returning to Pauli matrices (10), one can say that this result is the consequence of the fact that σ_3 is invariant to congruent unitary transformations. This also secures the fact that in backscatter if the **S** matrix is symmetric in one base it is symmetric in all bases (reciprocity theorem). In other words, the difference of the off-diagonal elements of the scattering matrix will be invariant to the radar polarization basis. Therefore, the parameter $\dot\Delta$ will only be determined by the non-reciprocal properties of radar object and can be considered as an objective characteristic of this object.

V. INSERT the text in the paragraph that was after Eqn. (26) in the previous version (Eqn. (30) in the corrected version with the renumbered equations), so that this paragraph takes the form:

It implies that the term $A_3\,\sigma_3$ in the decomposition (11) can be compared to a target which orthogonalises all incident polarisations (see, for example [17]). Thus, such target (by definition) will not take part in copolar RCS. That means that the signal scattered by a non-reciprocal object and received in a single-channel system will depend only on the «symmetric» part of the object's scattering matrix

Comment 3

3) The most important issue is the relation of this work to SVD. The basic problem is that for bistatic systems the maximum RCS will not be obtained for copolarised antennas as I showed in my first review.. the authors make no mention of the importance of this 'mixed' antenna optimisation. The main contribution of this paper is in bistatic systems and so to miss out such a basic observation is I think poor.
I think your paper in this form is misleading. Readers may get the impression that they can calculate maximum RCS from the Huynen parameters.this is not true for bistatic systems.

In order to answer this remark, the authors:

I. INSERT the following paragraph before the point "Orientation angle" with Eqn. (45) in the previous version (Eqn. (49) in the corrected version with the renumbered equations)

It should be noted that "m" parameter ("maximal polarization") would unambiguously characterize a non-reciprocal object for monostatic case. However, this value cannot be considered as the maximum response in the general case of bistatic configuration. Therefore, this parameter ("m") should be used with care in the latter case.

The authors would like to the following remark with respect to the 3rd comment

The contents of the paper do not include aspects of bi-static radar. The basic idea of the paper is to estimate the polarization properties of an object with asymmetrical scattering matrix by measuring the scattering matrix quadratures. That is why we mentioned monostatic case and linear basis. In our paper the "maximum polarization" parameter is similar to the S_{KK} parameter in [19], which considers the bistatic problem from the beginning.