Algorithms for Estimating the Complete Group of Polarization Invariants of the Scattering Matrix (SM) Based on Measuring All SM Elements

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***Abstract*—The procedure for estimating polarization invariants of the backscattering matrix in horizontal–vertical basis is considered for radar observation of arbitrary nonreciprocal objects. Two polarization invariants are added to the well-known six Huynen–Euler invariants. These new invariants (nonreciprocity angle and difference in absolute phases of the symmetric and antisymmetric parts of the scattering matrix) describe the nonreciprocal properties of the object itself. With the simultaneous measurement of all eight quadratures of the scattering matrix elements, the closed-form expressions for calculating the eight polarization invariants are given. The derived expressions are the starting point for complete estimation of the polarization properties of arbitrary radar objects with a nonsymmetric scattering matrix. The given approach can be used to study various polarization effects in remote radar sensing of artificial and natural objects, and also to simulate polarization measurement processes and estimation errors caused by the measurements of scattering matrix elements at different instants.**

***Index Terms*—Asymmetric backscattering matrix, Huynen–Euler parameters, horizontal–vertical basis, nonreciprocal object, nonreciprocity parameters, polarization invariants, radar polarimetry, scattering matrix elements quadratures.**

I. Introduction

THERE EXITS Western literature in which the research on principles of radar polarimetry and optimization procedures [11], **au: [11] is IEEE Symposium?** optimum polarizations finding [8]–[10], **au: issue no. and/or month of [8]?** and research of null polarizations of radar objects [12] are extended to bistatic cases or to monostatic radar, for measuring nonreciprocal objects. Papers of Russian polarimetrists also discuss results of investigations in the field of monostatic radar of nonreciprocal objects. With respect to the different aspects of optimum parametrization of asymmetrical scattering matrices for monostatic radar, an analysis of such a polarization characteristic of an arbitrary radar object (medium) as "nonreciprocity factor" was considered in Russia by Khlusov [13], [14]. The physical aspects of a backscattering mechanism by partly nonreciprocal objects were analyzed in [15] **au: publisher of [15]?** and [16]. In particular, an experimental testbed that simulates the radar channel with arbitrary polarization properties was described in [15]. In this paper, the application capabilities of a controllable radar reflector with partly nonreciprocal properties were also estimated. The analysis of bistatic radar polarimetry was made, and two new Euler angles, which determine the bistatic scattering matrix, were introduced in [19].

Evidently, the paper of Boerner *et al.* [1] should be considered as one of the first mentioning the scattering matrix (SM) measurement of a radar object with nonreciprocal polarization properties. The inequality of the measured off-axis scattering matrix elements for some regions in an electric storm has been explained by the presence of particle formation. Another experimental confirmation of the existence of nonreciprocal objects has been considered in [15], in which the design of an absolutely nonreciprocal reflector was described for the first time. Moreover, this paper includes the analysis of possible remote detection of spatial regions or ground surface areas where magnetic field and/or para- or ferromagnetics are present.

It is known that a polarimetric analysis of radar objects having arbitrary polarization properties can be made by means of coherent decomposition theorems. The Pauli spin matrices σ_0, σ_1, σ_2, and σ_3 approach for decomposition of an asymmetrical scattering matrix is widely used (see [11]). In this case, the backscattering matrix can be presented in the form

$$\mathbf{S} = \begin{bmatrix} a+b & c-jd \\ c+jd & a+b \end{bmatrix} = a\cdot\sigma_0 + b\cdot\sigma_1 + c\cdot\sigma_2 + d\cdot\sigma_3 \quad (1)$$

where a, b, c, and d are all complex values.

The investigation of radar objects in relation to possible backscattering mechanisms allows one to connect these values with four elementary deterministic point targets. In such a case, a, b, c, and d correspond to a sphere or plane surface, a diplane, a diplane rotated over 45°, and a scatterer that transforms every incident polarization into its orthogonal state, correspondingly.

In the Cameron approach [18], the scattering matrix is also decomposed using the Pauli matrices. In this case, the matrix is first decomposed into reciprocal and nonreciprocal components, and then the reciprocal component is decomposed into two further components, both of which have linear eigenpolarizations.

Khlusov [14] decomposes an arbitrary scattering matrix with the use of the Pauli matrices, reducing the matrix into the sum of a symmetrical matrix and an antisymmetric matrix weighted by a complex factor. With this approach, a radar object is described

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by a set of invariant parameters $(\dot{\lambda}_1, \dot{\lambda}_2, \theta, \tau)$, which characterize the symmetrical component, as well as by the so-called polarization nonreciprocity factor $\dot{\xi}$ that describes the nonreciprocal properties of the objects. The modulus of $\dot{\xi}$ is considered to be the ratio between the radar cross section (RCS) of the nonreciprocal part and the RCS of the whole object, while $\arg\{\dot{\xi}\}$ is interpreted as a spatial diversity of the reciprocal and nonreciprocal parts of the scattering matrix (SM).

From the results of theoretical studies and experimental measurements of the SM, one can conclude that the SM elements $\dot{S}_{ij}$ depend on many factors not connected with the scattering properties of observed radar objects.

1) choice of polarization basis (linear, circular, elliptical) realized in the radar system;
2) mutual orientation of radar and object;
3) spatial diversity between radar system and object, etc.

For this reason, the measurement and investigation of polarization invariants of the scattering matrix is fundamental to the theory and practice of radar polarimetry. These invariants are measurable values that characterize the polarization properties of radar objects themselves and do not depend on the polarization basis implemented in the radar system. Some polarization invariants measured in Russia and in the West have proven to contain much information about various radar targets.

In this paper, we consider the estimation of polarization invariants for the general case of an asymmetric scattering matrix supposing that all quadratures of the SM elements are known. In Section II we give a short review of the traditional polarization invariants. Then, we analyze the asymmetric matrix case in Section III. In Section IV we present the nonreciprocity parameters supplementing the Huynen–Euler invariants group. Section V is devoted to the derivation of the closed-form expressions for the complete group of eight invariants of the scattering matrix.

II. Huynen–Euler Polarization Invariants

As they describe the polarization properties of reciprocal radar objects and media (within the framework of the Sinclair scattering matrix concept), the Huynen–Euler group invariants (e.g., see [6]) **au: please cite [7] where applicable** are widely used. Their main advantage is that the given invariants group may be used not only for the estimation of the polarization properties of stable targets, but also for time-fluctuating random objects. In the latter case, it is quite acceptable to assume that the time correlation of the SM elements of fluctuating objects is much longer than the carrier period of the radiated signal. This allows us to speak about an opportunity of $\ll$ instant $\gg$ reduction of the scattering matrix to a diagonal form [3].

It should be noted that the given problem could be solved for the more general case of mixed bases, when TX and RX polarizations are different. Without loss of generality, we assume that the measuring polarization basis of the radar system is linear. Besides, the radar coordinate system coincides with Cartesian coordinates XOY. The scattering matrix of a radar object can then be written in the radar's polarization basis as

$$\mathbf{S} = \mathbf{T}^T \cdot \mathbf{E}^T \cdot \mathbf{S}_\mathrm{e} \cdot \mathbf{E} \cdot \mathbf{T} \cdot \mathrm{e}^{j\phi} = \begin{bmatrix} \dot{S}_{11} & \dot{S}_{12} \\ \dot{S}_{21} & \dot{S}_{22} \end{bmatrix} \tag{2}$$

where

$$\mathbf{S}_\mathrm{e} = \begin{bmatrix} \dot{\lambda}_1 & 0 \\ 0 & \dot{\lambda}_2 \end{bmatrix} \tag{3}$$

is the radar object's scattering matrix in the eigenpolarization basis;$\dot{\lambda}_1$, $\dot{\lambda}_2$ are complex eigenvalues of the scattering matrix

$$\mathbf{T} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} \cos(\varepsilon) & -j\sin(\varepsilon) \\ -j\sin(\varepsilon) & \cos(\varepsilon) \end{bmatrix} \tag{4}$$

are transformation matrices from the object's eigenbasis in the linear basis of the radar [4]; superscript T denotes the transposition of a matrix.

The scattering matrix can be written as [6]

$$\mathbf{S}_\mathrm{e} = m \cdot \mathrm{e}^{j\phi} \cdot \begin{bmatrix} \mathrm{e}^{j\nu} & 0 \\ 0 & \mathrm{e}^{-j\nu} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & \tan^2\gamma \end{bmatrix} \cdot \begin{bmatrix} \mathrm{e}^{j\nu} & 0 \\ 0 & \mathrm{e}^{-j\nu} \end{bmatrix} \tag{5}$$

so that the complex eigenvalues take the form

$$\dot{\lambda}_1 = m \cdot \mathrm{e}^{j(2\nu+\phi)} \quad \dot{\lambda}_2 = m \cdot \tan^2\gamma \cdot \mathrm{e}^{-j(2\nu-\phi)}. \tag{6}$$

Therefore, the symmetric scattering matrix can be described by six independent Huynen–Euler parameters:

m — *maximum polarization:* this value is the largest possible response from the radar target, and this response is received under transmission of the optimum polarization connected with the largest eigenvalue, i.e., the "m" value is equal to modulus of the first eigenvalue;

ϕ — absolute phase of the scattering matrix $(-180^\circ \le \phi < 180^\circ)$;

θ — orientation angle of the eigenbasis of the object relative to the radar coordinate system $(-90^\circ \le \theta < 90^\circ)$;

ε — ellipticity angle of the eigenbasis of the object $(-45^\circ \le \varepsilon < 45^\circ)$;

ν — skip angle $(-45^\circ \le \nu < 45^\circ)$; $4\nu = \arg\{\dot{\lambda}_1\} - \arg\{\dot{\lambda}_2\}$. Note that when the response from the object is caused by scattering mechanisms with an even number of reflections (bounces), the parameter ν is equal to 45°. In Russian literature on polarimetry, the "phase shift" value $\Delta\varphi = \arg\{\dot{\lambda}_1\} - \arg\{\dot{\lambda}_2\}$ (equal to the argument difference of the complex eigenvalues) is used instead of the ν parameter;

γ — characteristic angle $(0^\circ \le \gamma \le 45^\circ)$ or the "polarizability" angle; radar targets with $\gamma = 45^\circ$ do not change the polarization of the transmitted signal, whereas targets with $\gamma = 0^\circ$ will completely determine the polarization state of the reflected signal.

The angles θ and ε determine the ellipticity and orientation of the larger axis of the polarization ellipse of the electromagnetic wave. If a radar object is irradiated by the wave with such parameters, then the signal power received in a single-channel radar will be maximal. This means that the reflection factors for the transmission case of two orthogonal waves with the ellipticity angles ε and $-\varepsilon$ and orientation angles θ and $\theta \pm \pi/2$, respectively, will be proportional (to a precision of phase factor) to the eigenvalues $\dot{\lambda}_1$, $\dot{\lambda}_2$ of the object's scattering matrix.

III. Asymmetric Matrix Case

For monostatic radar, the backscattering matrix is considered to be symmetric and gives equality of its off-diagonal elements $\dot{S}_{12} = \dot{S}_{21}$. As a rule, the asymmetry of the scattering matrix becomes apparent in bistatic configurations. However, there are experimental data that prove that the scattering matrix may be asymmetric ($\dot{S}_{12} \neq \dot{S}_{21}$) in the monostatic case too [1]. In particular, this may occur in cases in which strong magnetic or electric field strengths caused by exterior energy sources are present in a bounded spatial volume.

The possibility of the existence of real objects, whose polarization properties are described by asymmetric ($\dot{S}_{12} \neq \dot{S}_{21}$) scattering matrices in the monostatic radar sensing case or in a bistatic configuration, allows us to widen the mentioned group of Huynen–Euler parameters by additional independent polarization invariants.

It is known that an asymmetric scattering matrix cannot be diagonalized by the following congruent transformation:

$$\mathbf{S} \rightarrow \mathbf{U}^T \cdot \mathbf{S} \cdot \mathbf{U} = \mathbf{S}_e = \mathrm{diag}[\dot{\lambda}_1, \dot{\lambda}_2] \tag{7}$$

which is typical for symmetric SMs. Here the matrix

$$\mathbf{U} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} \cos(\varepsilon) & j\sin(\varepsilon) \\ j\sin(\varepsilon) & \cos(\varepsilon) \end{bmatrix} \tag{8}$$

is the unitary unimodular transformation matrix [5] according to Takagi factorization.

In order to set up the complete group of polarization invariants, we start from the consideration that the scattering matrix of an arbitrary radar object in the Cartesian basis is known and presented by four complex values $\dot{S}_{ij}$

$$\mathbf{S} = \begin{bmatrix} \dot{S}_{11} & \dot{S}_{12} \\ \dot{S}_{21} & \dot{S}_{22} \end{bmatrix}. \tag{9}$$

In this case, the inequality $\dot{S}_{12} \neq \dot{S}_{21}$ is considered as the manifestation of the object's nonreciprocal properties.

We now decompose $\mathbf{S}$ by using the orthogonal system of Pauli matrices

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$\sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix} \tag{10}$$

so that the matrix (9) takes the form

$$\mathbf{S} = A_0\sigma_0 + A_1\sigma_1 + A_2\sigma_2 + A_3\sigma_3 = \sum_{i=0}^{3} A_i\sigma_i \tag{11}$$

where

$$A_i = 0.5 \cdot \mathrm{Sp}[\mathbf{S}\,\sigma_i]. \tag{12}$$

The scattering matrix $\mathbf{S}$ can be represented as

$$\mathbf{S} = 0.5\left\{(\dot{S}_{11} + \dot{S}_{22})\sigma_0 + (\dot{S}_{11} - \dot{S}_{22})\sigma_1 + (\dot{S}_{12} + \dot{S}_{21})\sigma_2 + (\dot{S}_{21} - \dot{S}_{12})\sigma_3\right\}. \tag{13}$$

The first three terms of the decomposition (13) describe the symmetric component $\mathbf{S}^{(s)}$

$$\mathbf{S}^{(s)} = \begin{bmatrix} \dot{S}_{11} & 0.5 \cdot (\dot{S}_{12} + \dot{S}_{21}) \\ 0.5 \cdot (\dot{S}_{12} + \dot{S}_{21}) & \dot{S}_{22} \end{bmatrix} \tag{14}$$

and the fourth term the skew-symmetric component

$$\mathbf{S}^{(a)} = \begin{bmatrix} 0 & -j0.5 \cdot (\dot{S}_{21} - \dot{S}_{12}) \\ j0.5 \cdot (\dot{S}_{21} - \dot{S}_{12}) & 0 \end{bmatrix} = j\dot{\Delta} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{15}$$

where

$$\dot{\Delta} = 0.5 \cdot (\dot{S}_{21} + \dot{S}_{12}). \tag{16}$$

As a result, the original matrix is written as

$$\mathbf{S} = \mathbf{S}^{(s)} + \mathbf{S}^{(a)}. \tag{17}$$

Since the "j" factor in (15) is not important in our further analysis, it will be omitted hereinafter.

The scattering matrix (9) in the polarization basis with parameters ε', θ' becomes

$$\mathbf{S}' = \mathbf{U}'^T \cdot \mathbf{S} \cdot \mathbf{U}' \tag{18}$$

with the transformation matrix

$$\mathbf{U}' = \begin{bmatrix} \cos(\theta') & -\sin(\theta') \\ \sin(\theta') & \cos(\theta') \end{bmatrix} \cdot \begin{bmatrix} \cos(\varepsilon') & j\sin(\varepsilon') \\ j\sin(\varepsilon') & \cos(\varepsilon') \end{bmatrix}. \tag{19}$$

Substituting (17) in (18), we obtain

$$\mathbf{S}' = \mathbf{U}'^T \cdot \left(\mathbf{S}^{(s)} + \mathbf{S}^{(a)}\right) \cdot \mathbf{U}' = \mathbf{U}'^T \cdot \mathbf{S}^{(s)} \cdot \mathbf{U}' + \mathbf{U}'^T \cdot \mathbf{S}^{(a)} \cdot \mathbf{U}' \tag{20}$$

and

$$\mathbf{U}'^T \cdot \mathbf{S}^{(\mathrm{a})} \cdot \mathbf{U}' = \dot{\Delta} \cdot \left\{\mathbf{U}'^T \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \mathbf{U}'\right\}.$$

One can show that for any unitary transformation matrix (19) the following equality exists:

$$\mathbf{U}'^T \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \mathbf{U}' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{21}$$

Therefore, (20) may be rewritten in the general form as

$$\mathbf{S}' = \mathbf{U}'^T \cdot \mathbf{S}^{(\mathrm{s})} \cdot \mathbf{U}' + \dot{\Delta} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{22}$$

Since the ellipticity and orientation angles are chosen arbitrarily, it is possible to conclude that the second item in (22) with the proportional factor $\dot{\Delta} = 0.5 \cdot (\dot{S}_{21} - \dot{S}_{12})$ will not depend on ε', θ' parameters in the transformation matrix $\mathbf{U}'$. With reference to the Pauli matrices (10), one can say that this result is the consequence of the fact that σ_3 is invariant to congruent unitary transformations. This also secures the condition that in backscatter when the $\mathbf{S}$ matrix is symmetric in one base, it is symmetric in all bases (reciprocity theorem). In other words, the difference of the off-diagonal elements of the scattering matrix will be invariant to the radar polarization basis. Therefore, the parameter $\dot{\Delta}$ will only be determined by the nonreciprocal properties of radar object and can be considered as an objective characteristic of this object.

IV. Nonreciprocity Parameters

Suppose that the unitary matrix $\mathbf{U}' = \mathbf{U}_0$ and the given values of the ellipticity angle ε' and orientation angle θ' coincide with the eigenpolarization basis (ε_0, θ_0) parameters of the symmetric part $\mathbf{S}^{(s)}$ of the scattering matrix. In this case, the matrix $\mathbf{S}^{(s)}$ is diagonalized into

$$\mathbf{U}_0^T \cdot \mathbf{S}^{(s)} \cdot \mathbf{U}_0 = \begin{bmatrix} \dot{\lambda}_{01} & 0 \\ 0 & \dot{\lambda}_{02} \end{bmatrix} \tag{23}$$

and the initial (asymmetric) scattering matrix in this basis can be presented as

$$\mathbf{S}_0 = \mathbf{U}_0^T \cdot \left(\mathbf{S}^{(s)} + \mathbf{S}^{(a)}\right) \cdot \mathbf{U}_0 = \begin{bmatrix} \dot{\lambda}_{01} & 0 \\ 0 & \dot{\lambda}_{02} \end{bmatrix} + \dot{\Delta} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{24}$$

Let unit vectors $\vec{h}$ and $\vec{x}$ denote the polarization states of the receiving and transmitting antennas, respectively, with $\|\vec{h}\| = \|\vec{x}\| = 1$. Then, the received signal voltage at the receive antenna is written as

$$\dot{V}_{\mathrm{r}} = \vec{h} \cdot \mathbf{S}\vec{x} \equiv \vec{h}^T \cdot \mathbf{S}\vec{x} \equiv (\vec{h}, \mathbf{S}\vec{x}). \tag{25}$$

In a monostatic radar, the same antenna is used for radiation and reception of radar signals. Expression (25) can be rewritten into

$$\dot{V}_{\mathrm{r}} = \vec{h}^T \cdot \mathbf{S}\vec{h}. \tag{26}$$

In this case, the normalized power transfer equation in a single-channel monostatic system becomes

$$P = |\vec{h}^T \cdot \mathbf{S}\vec{h}|^2. \tag{27}$$

Vector $\vec{h}$ is represented as

$$\vec{h} = \mathbf{U}'' \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{28}$$

with transformation matrix

$$\mathbf{U}'' = \begin{bmatrix} \cos(\theta'') & -\sin(\theta'') \\ \sin(\theta'') & \cos(\theta'') \end{bmatrix} \cdot \begin{bmatrix} \cos(\varepsilon'') & j\sin(\varepsilon'') \\ j\sin(\varepsilon'') & \cos(\varepsilon'') \end{bmatrix}.$$

The signal scattered by a nonreciprocal object with scattering matrix (9) and received in the single-channel system becomes $\dot{V}_{\mathrm{r}} = \vec{h}^T \cdot \mathbf{S}\vec{h} = [1\ \ 0] \cdot \mathbf{U}''^T \cdot (\mathbf{S}^{(s)} + \mathbf{S}^{(a)}) \cdot \mathbf{U}'' \cdot \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]$. By taking (15) into consideration, we derive

$$\dot{V}_{\mathrm{r}} = [1 \quad 0] \cdot \mathbf{U}''^T \cdot \mathbf{S}^{(s)} \cdot \mathbf{U}'' \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \dot{\Delta} \cdot \left\{ [1 \quad 0] \cdot \mathbf{U}''^T \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \mathbf{U}'' \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}. \tag{29}$$

For the last term in (29), it can easily be shown that for any ε'', θ'' values

$$[1 \quad 0] \cdot \mathbf{U}''^T \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \mathbf{U}'' \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0. \tag{30}$$

It implies that the term $A_3\sigma_3$ in decomposition (11) describes a target that orthogonalizes all incident polarizations (e.g., see [17]). Such target (by definition) will not take part in copolar RCS investigations. That means that the signal scattered by a nonreciprocal object and received in a single-channel system will depend only on the $\ll$ symmetric $\gg$ part of the object's scattering matrix

$$\dot{V}_{\mathrm{r}} = [1 \quad 0] \cdot \mathbf{U}''^T \cdot \mathbf{S}^{(s)} \cdot \mathbf{U}'' \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{31}$$

The value $\dot{V}_{\mathrm{r}}$ will give maximum values (in power) only when the polarization state of the transmitting–receiving antenna coincides with the eigenpolarizations of the symmetric part $\mathbf{S}^{(s)}$, $\vec{h} = \vec{h}_0$ where

$$\vec{h}_0 = \mathbf{U}_0 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta_0) & -\sin(\theta_0) \\ \sin(\theta_0) & \cos(\theta_0) \end{bmatrix} \cdot \begin{bmatrix} \cos(\varepsilon_0) & j\sin(\varepsilon_0) \\ j\sin(\varepsilon_0) & \cos(\varepsilon_0) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{32}$$

and ε_0, θ_0 are the ellipticity and orientation angles of the eigenbasis of the matrix $\mathbf{S}^{(s)}$, respectively. In other words, the polarization invariants $\varepsilon_0, \theta_0, \dot{\lambda}_{01}, \dot{\lambda}_{02}$ (i.e., six Huynen–Euler invariants) will play the same role in the description of a nonreciprocal object as in that of a general reciprocal case.

For the description of the nonreciprocal properties of radar objects with an asymmetric scattering matrix, Khlusov [14] introduced the complex nonreciprocity factor

$$\dot{\xi} = \frac{\sqrt{2}\dot{\Delta}}{\|\mathbf{S}\|} \tag{33}$$

where $\|\mathbf{S}\|$ is the Euclidean norm of the scattering matrix (9) $\|\mathbf{S}\| = [\sum_{i=1}^{2}\sum_{j=1}^{2}(S_{ij} \cdot S_{ij})]^{0.5}$. Since the difference of the off-diagonal elements ($\dot{\Delta}$) and $\|\mathbf{S}\|$ does not depend on the polarization basis, the $\dot{\xi}$ value is also polarization invariant and will be only determined by the nonreciprocal properties of the radar object.

It is not difficult to show that the following equality follows from expression (21):

$$\mathrm{span}(\mathbf{S}) = \mathrm{span}\left(\mathbf{S}^{(s)}\right) + \mathrm{span}\left(\mathbf{S}^{(a)}\right) \tag{34}$$

where

$$\begin{aligned} \mathrm{span}(\mathbf{S}) &= |\dot{S}_{11}|^2 + |\dot{S}_{12}|^2 + |\dot{S}_{21}|^2 + |\dot{S}_{22}|^2 \\ \mathrm{span}\left(\mathbf{S}^{(s)}\right) &= |\dot{S}_{11}|^2 + |\dot{S}_{22}|^2 + 0.5 \cdot |\dot{S}_{12} + \dot{S}_{21}|^2 \\ \mathrm{span}\left(\mathbf{S}^{(a)}\right) &= 0.5 \cdot |\dot{S}_{21} - \dot{S}_{12}|^2. \end{aligned}$$

We now define the squared norm ratio of the antisymmetric part and the same value for the matrix $\mathbf{S}$ itself, as

$$\frac{\mathrm{span}\left(\mathbf{S}^{(a)}\right)}{\mathrm{span}(\mathbf{S})} = \frac{\left\|\mathbf{S}^{(a)}\right\|^2}{\|\mathbf{S}\|^2} = \frac{0.5 \cdot |\dot{S}_{21} - \dot{S}_{12}|^2}{|\dot{S}_{11}|^2 + |\dot{S}_{12}|^2 + |\dot{S}_{21}|^2 + |\dot{S}_{22}|^2}. \tag{35}$$

With $\mathrm{span}(\mathbf{S}^{(a)}) = 2 \cdot |\dot{\Delta}|^2$, expression (35) can be rewritten in the form, $\mathrm{span}(\mathbf{S}^{(a)})/\mathrm{span}(\mathbf{S}) = (2 \cdot |\dot{\Delta}|^2/\|\mathbf{S}\|^2)$, and we learn that

$$|\dot{\xi}|^2 = \frac{2 \cdot |\dot{\Delta}|^2}{\|\mathbf{S}\|^2}. \tag{36}$$

The physical meaning of the nonreciprocity factor $\dot{\xi}$ is that the squared modulus of this value contains information on the ratio between the RCS of the nonreciprocal part and the full RCS of the radar object. It is obvious that for all reciprocal objects $\dot{\xi} = 0$, while for partially nonreciprocal objects $|\dot{\xi}| \in (0;1)$.

In the general case, the nonreciprocity factor value is complex. One possible interpretation of $\{\dot{\xi}\}$ is given in [14]. The author considers this value as the difference between the absolute phases of the symmetric and antisymmetric parts of the scattering matrix. This value can be represented as a spatial diversity of "reciprocal" and "nonreciprocal" parts of the scattering matrix, just as $\arg\{\dot{\lambda}_1\} - \arg\{\dot{\lambda}_2\}$ of the symmetric scattering matrix can be treated as spatial diversity of orthogonal dipoles along the line of sight in so called "two-dipole model" [2] of a radar object.

Since the complex factor $\dot{\xi}$ contains full information concerning the nonreciprocal properties of arbitrary radar objects, the group of six Huynen–Euler invariants may be supplemented by two additional polarization invariants, both having an angular dimension.

- ζ nonreciprocity angle $(0^\circ \le \zeta \le 45^\circ)$ is equal to the arctangent of the $\dot{\xi}$ modulus; the value $\zeta = 0^\circ$ describes the radar sensing of reciprocal objects with a symmetric scattering matrix, whereas the value $\zeta = 45^\circ$ describes objects that are completely nonreciprocal;
- η the difference in absolute phases of the symmetric and antisymmetric parts of the scattering matrix $(-180^\circ \le \eta < 180^\circ)$.

Thus, the nonreciprocity factor can be represented as

$$\dot{\xi} = \zeta \cdot e^{j\eta} = \mathrm{atan}\left(|\dot{\xi}|\right) \cdot e^{j\eta}. \tag{37}$$

From the above we note that the suggested complete group of eight polarization invariants describes the polarization properties of arbitrary radar objects in an optimum way. This group of invariants consists of

$$m, \phi, \theta, \varepsilon, \nu, \gamma, \zeta, \eta. \tag{38}$$

V. Complete Invariants Group

We assume that the radar polarization basis is linear and that its coordinates system coincides with the Cartesian system *XOY*. In this basis, the scattering matrix of a nonreciprocal radar object is written as

$$\mathbf{S}_{\mathrm{xy}} = \mathbf{T}^T \cdot \mathbf{E}^T \cdot \mathbf{S}_{\mathrm{e}} \cdot \mathbf{E} \cdot \mathbf{T} \cdot \mathrm{e}^{j\phi} = \begin{bmatrix} \dot{S}_{11} & \dot{S}_{12} \\ \dot{S}_{21} & \dot{S}_{22} \end{bmatrix} \tag{39}$$

where $\dot{S}_{12} \ne \dot{S}_{21}$.

We assume that all eight quadratures of the SM elements were simultaneously measured by the radar, meaning that

$$\begin{aligned} I_1 &= \mathrm{Re}\{\dot{S}_{11}\} \quad Q_1 = \mathrm{Im}\{\dot{S}_{11}\} \\ I_2 &= \mathrm{Re}\{\dot{S}_{22}\} \quad Q_2 = \mathrm{Im}\{\dot{S}_{22}\} \\ I_3 &= \mathrm{Re}\{\dot{S}_{12}\} \quad Q_3 = \mathrm{Im}\{\dot{S}_{12}\} \\ I_4 &= \mathrm{Re}\{\dot{S}_{21}\} \quad Q_4 = \mathrm{Im}\{\dot{S}_{21}\} \end{aligned} \tag{40}$$

where I_k is the in-phase component, and Q_k are the quadrature components $(k = 1, \dots, 4)$ of the corresponding $\dot{S}_{ij}$ elements. In such a case, we may find analytic forms for the complete invariants group (38). Therefore, we present the initial scattering matrix $\mathbf{S}_{\mathrm{xy}}$ as the sum of the symmetric and antisymmetric matrices

$$\mathbf{S}_{\mathrm{xy}} = \mathbf{S}_{\mathrm{xy}}^{(s)} + \mathbf{S}_{\mathrm{xy}}^{(a)} = \begin{bmatrix} \dot{S}_{11}^{(s)} & \dot{S}_{12}^{(s)} \\ \dot{S}_{21}^{(s)} & \dot{S}_{22}^{(s)} \end{bmatrix} + \begin{bmatrix} 0 & \dot{S}_{12}^{(a)} \\ \dot{S}_{21}^{(a)} & 0 \end{bmatrix} \tag{41}$$

where the corresponding elements of $\mathbf{S}_{\mathrm{xy}}^{(s)}$ and $\mathbf{S}_{\mathrm{xy}}^{(a)}$ are written as

$$\begin{aligned} \dot{S}_{11}^{(s)} &= \dot{S}_{11} \quad \dot{S}_{22}^{(s)} = \dot{S}_{22} \\ \dot{S}_{12}^{(s)} &= \dot{S}_{21}^{(s)} = 0.5 \cdot (\dot{S}_{12} + \dot{S}_{21}) \end{aligned} \tag{42}$$

$$\begin{aligned} \dot{S}_{12}^{(a)} &= -0.5 \cdot (\dot{S}_{21} - \dot{S}_{12}) \\ \dot{S}_{21}^{(a)} &= +0.5 \cdot (\dot{S}_{21} - \dot{S}_{12}). \end{aligned} \tag{43}$$

We first focus our attention to the analytic forms for the polarization invariants describing the properties of the symmetric part of the scattering matrix. Toward this end, we denote the $\mathbf{S}_{\mathrm{xy}}^{(s)}$ matrix quadratures as

$$\begin{aligned} I_1^{(s)} &= I_1 \quad Q_1^{(s)} = Q_1 \quad I_2^{(s)} = I_2 \quad Q_2^{(s)} = Q_2 \\ I_3^{(s)} &= 0.5 \cdot (I_3 + I_4), \qquad Q_3^{(s)} = 0.5 \cdot (Q_3 + Q_4). \end{aligned} \tag{44}$$

To find the solution, we need to present the elements of the symmetric part of the scattering matrix as functions of the Huynen–Euler invariants. In the expression

$$\mathbf{S}_{\mathrm{xy}}^{(s)} = \mathbf{T}_0^T \cdot \mathbf{E}_0^T \cdot \mathbf{S}_{\mathrm{e}} \cdot \mathbf{E}_0 \cdot \mathbf{T}_0 \cdot e^{j\phi_0} \tag{45}$$

we substitute the symmetric part of the scattering matrix in the eigenbasis of the object

$$\mathbf{S}_e^{(s)} = \begin{bmatrix} \dot{\lambda}_{01} & 0 \\ 0 & \dot{\lambda}_{02} \end{bmatrix}$$

for the transformation matrices

$$\mathbf{T}_0 = \begin{bmatrix} \cos(\theta_0) & -\sin(\theta_0) \\ \sin(\theta_0) & \cos(\theta_0) \end{bmatrix}, \ \mathbf{E}_0 = \begin{bmatrix} \cos(\varepsilon_0) & j\sin(\varepsilon_0) \\ j\sin(\varepsilon_0) & \cos(\varepsilon_0) \end{bmatrix}$$

and for the complex eigenvalues

$$\dot{\lambda}_{01} = |\dot{\lambda}_{01}| \cdot e^{j(2\nu_0 + \phi_0)}, \qquad \dot{\lambda}_{02} = |\dot{\lambda}_{02}| \cdot e^{-j(2\nu_0 - \phi_0)}$$

where $|\dot{\lambda}_{01}| = m_0$, $|\dot{\lambda}_{02}| = m_0 \cdot \tan^2\gamma_0$.

After these substitutions, the resultant expressions for the matrix $\mathbf{S}_{\mathrm{xy}}^{(s)}$ elements have a quite complicated form. To simplify the expressions, we introduce the following:

$$\begin{aligned} a &= \mathrm{Re}\{\dot{\lambda}_{01}\} = |\dot{\lambda}_{01}| \cdot \cos(\phi_0 + 2\nu_0) \\ b &= \mathrm{Im}\{\dot{\lambda}_{01}\} = |\dot{\lambda}_{01}| \cdot \sin(\phi_0 + 2\nu_0) \\ d &= \mathrm{Re}\{\dot{\lambda}_{02}\} = |\dot{\lambda}_{02}| \cdot \cos(\phi_0 - 2\nu_0) \\ f &= \mathrm{Im}\{\dot{\lambda}_{02}\} = |\dot{\lambda}_{02}| \cdot \sin(\phi_0 - 2\nu_0). \end{aligned}$$

We now can write for the in-phase and quadrature components

$$\begin{aligned} I_1^{(s)} &= 0.5 \cdot [(a-d) \cdot \cos(2\theta_0) + (a+d) \cdot \cos(2\varepsilon_0) \\ &\qquad -(b+f) \cdot \sin(2\theta_0) \cdot \sin(2\varepsilon_0)] \\ Q_1^{(s)} &= 0.5 \cdot [(b-f) \cdot \cos(2\theta_0) + (b+f) \cdot \cos(2\varepsilon_0) \\ &\qquad +(a+d) \cdot \sin(2\theta_0) \cdot \sin(2\varepsilon_0)] \\ I_2^{(s)} &= 0.5 \cdot [-(a-d) \cdot \cos(2\theta_0) + (a+d) \cdot \cos(2\varepsilon_0) \\ &\qquad +(b+f) \cdot \sin(2\theta_0) \cdot \sin(2\varepsilon_0)] \\ Q_2^{(s)} &= -0.5 \cdot [(b-f) \cdot \cos(2\theta_0) - (b+f) \cdot \cos(2\varepsilon_0) \\ &\qquad +(a+d) \cdot \sin(2\theta_0) \cdot \sin(2\varepsilon_0)] \\ I_3^{(s)} &= 0.5 \cdot [(a-d) \cdot \sin(2\theta_0) + (b+f) \cos(2\theta_0) \cdot \sin(2\varepsilon_0)] \\ Q_3^{(s)} &= 0.5 \cdot [(b-f) \cdot \sin(2\theta_0) - (a+d) \cos(2\theta_0) \cdot \sin(2\varepsilon_0)]. \end{aligned}$$

This set of six equations can be reduced to a more convenient form, in order to derive expressions for the invariants. For this purpose, we must find sums and differences of the real and imaginary parts of the diagonal elements, and also the doubled real and imaginary parts of the off-diagonal element of the symmetric matrix. After simple transformations, this new set of equations takes the form

$$\begin{cases} (a+d) \cdot \cos(2\varepsilon_0) = I_1^{(s)} + I_2^{(s)} \\ (b+f) \cdot \cos(2\varepsilon_0) = Q_1^{(s)} + Q_2^{(s)} \\ (a-d) \cdot \cos(2\theta_0) - (b+f) \cdot \sin(2\theta_0) \cdot \sin(2\varepsilon_0) = I_1^{(s)} - I_2^{(s)} \\ (b-f) \cdot \cos(2\theta_0) + (a+d) \cdot \sin(2\theta_0) \cdot \sin(2\varepsilon_0) = Q_1^{(s)} - Q_2^{(s)} \\ (a-d) \cdot \sin(2\theta_0) + (b+f) \cdot \cos(2\theta_0) \cdot \sin(2\varepsilon_0) = 2I_3^{(s)} \\ (b-f) \cdot \sin(2\theta_0) - (a+d) \cdot \cos(2\theta_0) \cdot \sin(2\varepsilon_0) = 2Q_3^{(s)}. \end{cases} \tag{46}$$

The main calculations for deriving the scattering matrix invariants as functions of the quadratures of the SM elements primarily use this set of equations and are given in the Appendix. Below we present only final expressions for eight polarization invariants.

Maximum polarization (m)

$$m_0 = |\dot{\lambda}_{01}| = \left[0.5 \cdot \left(\mathrm{span}\left(\mathbf{S}^{(s)}\right) + P\right)\right]^{0.5} \tag{47}$$

where

$$\begin{aligned} P &= \left[\mathrm{span}^2\left(\mathbf{S}^{(s)}\right) - 4 \cdot \det{}^2\left(\mathbf{S}^{(s)}\right)\right]^{0.5} \\ \mathrm{span}\left(\mathbf{S}^{(s)}\right) &= \left(I_1^2 + Q_1^2\right) + \left(I_2^2 + Q_2^2\right) + 0.5 \cdot (I_3 + I_4)^2 \\ &\quad + 0.5 \cdot (Q_3 + Q_4)^2 \\ \left|\det\left(\mathbf{S}^{(s)}\right)\right| &= \Big\{\left[(I_1 \cdot I_2 - Q_1 \cdot Q_2) - 0.25 \cdot (I_3 + I_4)^2 \right. \\ &\quad \left. +0.25 \cdot (Q_3 + Q_4)^2\right]^2 \\ &\quad + \left[(I_1 \cdot Q_2 + Q_1 \cdot I_2) - 0.5 \cdot (I_3 + I_4) \right. \\ &\quad \left. \cdot (Q_3 + Q_4)\right]^2\Big\}^{0.5} \end{aligned} \tag{48}$$

It should be noted that parameter "m" ("maximal polarization") would unambiguously characterize the nonreciprocal object for the monostatic case. However, this value cannot be considered as the maximum response in the general case of a bistatic configuration. Therefore, this parameter ("m") should be used with care in the latter case. Orientation angle (θ) is

$$\theta_0 = 0.5 \cdot \mathrm{atan}\left(\frac{(I_1+I_2) \cdot (I_3+I_4) + (Q_1+Q_2) + (Q_3+Q_4)}{(I_1^2+Q_1^2) - (I_2^2+Q_2^2)}\right). \tag{49}$$

Ellipticity angle (ε) is

$$\varepsilon_0 = 0.5 \cdot \mathrm{atan}\left(\frac{(Q_1-Q_2) \cdot \sin(2\theta_0) - (Q_3+Q_4) \cdot \cos(2\theta_0)}{I_1+I_2}\right). \tag{50}$$

Skip angle (ν) (under the restriction that $(1+\cos(4\varepsilon_0)) \cdot (1-\cos(4\theta_0)) \neq 0$) is

$$\nu_0 = \begin{cases} \nu_0^{(p)}, & D_a \cdot D_d > -D_b \cdot D_f \\ \nu_0^{(p)} + \frac{\pi}{4}, & D_a \cdot D_d \leq -D_b \cdot D_f, \frac{D_b}{D_a} > 0 \\ \nu_0^{(p)} - \frac{\pi}{4}, & D_a \cdot D_d \leq -D_b \cdot D_f, \frac{D_b}{D_a} < 0. \end{cases} \tag{51}$$

where **au: please add text to describe expressions and direct reader** (see equation at bottom of the next page). Characteristic angle (γ) is

$$\gamma_0 = \mathrm{atan}\left(\left(\frac{\mathrm{span}\left(\mathbf{S}^{(s)}\right) - P}{\mathrm{span}\left(\mathbf{S}^{(s)}\right) + P}\right)^{0.25}\right). \tag{52}$$

Absolute phase (ϕ) is

$$\phi_0 = \begin{cases} \phi_0^{(p)}, & G = +0.5 \cdot (I_3 + I_4) \\ \phi_0^{(p)} - \pi, & G = -0.5 \cdot (I_3 + I_4) \end{cases} \tag{53}$$

where

$$\begin{aligned} \phi_0^{(p)} &= 0.5 \\ &\cdot \mathrm{atan}\left(\frac{(I_1 \cdot Q_2 + Q_1 \cdot I_2) - 0.5 \cdot (I_3+I_4) \cdot (Q_3+Q_4)}{(I_1 \cdot I_2 - Q_1 \cdot Q_2) - 0.25 \cdot ((I_3+I_4)^2 - (Q_3+Q_4)^2)}\right) \end{aligned}$$

and

$$\begin{aligned} G &= F1 \cdot \cos\phi_0^{(p)} - F2 \cdot \sin\phi_0^{(p)} \\ F1 &= 0.5 \cdot m \cdot (1 - \tan^2\gamma_0) \\ &\quad \cdot [\cos(2\nu_0) \cdot \sin(2\theta_0 + \sin(2\nu_0) \cdot \cos(2\theta_0) \cdot \sin(2\varepsilon_0)] \\ F2 &= 0.5 \cdot m \cdot (1 + \tan^2\gamma_0) \\ &\quad \cdot [\sin(2\nu_0) \cdot \sin(2\theta_0 - \cos(2\nu_0) \cdot \cos(2\theta_0) \cdot \sin(2\varepsilon_0)]. \end{aligned}$$

Nonreciprocity angle (ζ) is

$$\zeta = \mathrm{atan}\left(\frac{\sqrt{2}}{2} \cdot \left(\frac{(I_4-I_3)^2 + (Q_4-Q_3)^2}{(I_1^2+Q_1^2) + (I_2^2+Q_2^2) + (I_3^2+Q_3^2) + (I_4^2+Q_4^2)}\right)^{0.5}\right). \tag{54}$$

Difference of absolute phases (η)

$$\eta = \arg\{\dot{\xi}\} = \operatorname{atan}\left(\frac{Q_4 - Q_3}{I_4 - I_3}\right). \tag{55}$$

It is possible to illustrate the results above with an example. Let the scattering matrix of a radar target have the form

$$\mathbf{S} = \begin{bmatrix} 0.5 + j0.3 & 0.4 - j0.19 \\ 0.2 + j0.16 & 0.2 + j0.6 \end{bmatrix}.$$

In this case, symmetric and a skew-symmetric components of the scattering matrix are written as

$$\mathbf{S}^{(s)} = \begin{bmatrix} 0.5 + j0.3 & 0.3 - j0.015 \\ 0.3 + j0.015 & 0.2 + j0.6 \end{bmatrix}$$
$$\mathbf{S}^{(a)} = \begin{bmatrix} 0 & 0.1 - j0.175 \\ -0.1 + j0.175 & 0 \end{bmatrix}.$$

Using (47)–(53), we find the following values of the Huynen–Euler invariants, which characterize the SM symmetric component

$$\mathrm{m} = 0.823 \quad \theta = 49.34^\circ \quad \varepsilon = -11.637^\circ$$
$$\nu = -10.061^\circ \quad \gamma = 37.769^\circ \quad \phi = 57.353^\circ.$$

Finally, we calculate [by (54) and (55)] the nonreciprocity angle and difference of absolute phases, which describe nonreciprocal properties of the radar target

$$\zeta = 15.897^\circ \quad \eta = 60.255^\circ.$$

Thus, we obtain a complete group of eight polarization invariants describing an arbitrary radar target.

VI. Conclusions

It has been shown that the complete group of polarization invariants can be estimated by using the results of the eight simultaneously measured quadrature components of the scattering matrix elements. The suggested group of parameters is based on the Huynen–Euler invariants supplemented with two additional invariants describing the nonreciprocal properties of an arbitrary radar object. The derived expressions (47) and (49)–(55) form the algorithmic basis for estimating the complete group of polarization invariants. It should be noted that seven invariants have an angular dimension and, therefore, are very convenient for the analysis and comparison of radar data. Unfortunately, the authors could not present an example of using their approach to real polarimetric measurements or SAR imagery. The main reason is the absence of representative data of simultaneous measurement of all elements of the scattering matrix. The authors intend to use this approach for investigating the polarization properties of radar objects in the framework of future joint research between IRCTR TUDelft (The Netherlands) and TUCSR (Russia). It is possible to apply the derived expressions of polarization invariants in statistical simulation and also for evaluating errors caused by nonsimultaneous measurement of SM elements.

Appendix

A. Maximum Polarization

It is known (e.g., see [3]) that the sum of squared moduli of the symmetric matrix elements

$$\operatorname{span}\left(\mathbf{S}^{(s)}\right) = \left|\dot{S}_{11}^{(s)}\right|^2 + \left|\dot{S}_{22}^{(s)}\right|^2 + 2\cdot\left|\dot{S}_{12}^{(s)}\right|^2 = |\dot{\lambda}_{01}|^2 + |\dot{\lambda}_{02}|^2 \tag{A.1}$$

and the determinant modulus

$$\left|\det\left(\mathbf{S}^{(s)}\right)\right| = \left|\dot{S}_{11}^{(s)}\cdot\dot{S}_{22}^{(s)} - \left(\dot{S}_{12}^{(s)}\right)^2\right| = |\dot{\lambda}_{01}|\cdot|\dot{\lambda}_{02}| \tag{A.2}$$

are polarization invariants that do not depend on the radar basis.

Using these expressions, we find the maximum polarization value as

$$m_0 = |\dot{\lambda}_{01}| = \left[0.5\cdot\left(\operatorname{span}\left(\mathbf{S}^{(s)}\right) + P\right)\right]^{0.5} \tag{A.3}$$

where

$$P = \left[\operatorname{span}^2\left(\mathbf{S}^{(s)}\right) - 4\cdot\det^2\left(\mathbf{S}^{(s)}\right)\right]^{0.5}. \tag{A.4}$$

$$D_a = V1 + (V2 - V3) \quad D_b = U1 - (U2 - U3) \quad D_d = V1 - (V2 - V3) \quad D_f = U1 + (U2 + U3)$$
$$V1 = (I_1 + I_2)\cdot\cos(2\varepsilon_0)\cdot(1 - \cos(4\theta_0)) \quad U1 = (Q_1 + Q_2)\cdot\cos(2\varepsilon_0)\cdot(1 - \cos(4\theta_0))$$
$$V2 = 0.5\cdot(Q_1 + Q_2)\cdot\sin(4\theta_0)\cdot\sin(4\varepsilon_0) \quad U2 = 0.5\cdot(I_1 + I_2)\cdot\sin(4\theta_0)\cdot\sin(4\varepsilon_0)$$
$$V3 = (I_3 + I_4)\cdot\sin(2\theta_0)\cdot(1 + \cos(4\varepsilon_0)) \quad U3 = (Q_3 + Q_4)\cdot\sin(2\theta_0)\cdot(1 + \cos(4\varepsilon_0))$$
$$\nu_0^{(p)} = 0.25\cdot\operatorname{atan}\left(\frac{0.5\cdot P2\cdot[(I_3 + I_3)\cdot(Q_1 + Q_2) - (Q_3 + Q_4)\cdot(I_1 + I_2)]}{P4\cdot[(I_1 + I_2)^2 + (Q_1 + Q_2)^2] + 0.5\cdot P3\cdot[(I_3 + I_4)\cdot(Q_1 + Q_2) - (Q_3 + Q_4)\cdot(I_1 + I_2)]} \right.$$
$$\left. \frac{-P1\cdot[(I_1 + I_2)^2 + (Q_1 + Q_2)^2]}{-0.25\cdot P5\cdot[(I_3 + I_4)^2 + (Q_3 + Q_4)^2]}\right)$$
$$P1 = \sin(4\theta_0)\cdot(1 - \cos(4\theta_0))\cdot\sin(4\varepsilon_0)\cdot\cos(2\varepsilon_0) \qquad P2 = 4\cdot\sin(2\theta_0)\cdot(1 - \cos(4\theta_0))\cdot\cos(2\varepsilon_0)\cdot(1 + \cos(4\varepsilon_0))$$
$$P3 = 2\cdot\sin(2\theta_0)\cdot\sin(4\theta_0)\cdot\sin(4\varepsilon_0)\cdot(1 + \cos(4\varepsilon_0)) \quad P4 = \cos^2(2\varepsilon_0)\cdot(1 - \cos(4\theta_0))^2 - 0.25\cdot\sin^2(4\theta_0)\cdot\sin^2(4\varepsilon_0)$$
$$P5 = 4\cdot\sin^2(2\theta_0)\cdot(1 + \cos(4\varepsilon_0))^2.$$

It is obvious that the "span" (A.1) and the squared modulus of the determinant (A.2), which are used in "m," are functions of the initial SM element quadratures via

$$\begin{aligned}\mathrm{span}\left(\mathbf{S}^{(s)}\right) &= \left(I_1^2+Q_1^2\right)+\left(I_2^2+Q_2^2\right)\\ &\quad +0.5\cdot(I_3+I_4)^2+0.5\cdot(Q_3+Q_4)^2 \quad \text{(A.5)}\\ \left|\det\left(\mathbf{S}^{(s)}\right)\right| &= \Big[\big[(I_1\cdot I_2-Q_1\cdot Q_2)-0.25\\ &\quad \cdot(I_3+I_4)^2+0.25\cdot(Q_3+Q_4)^2\big]^2\\ &\quad +\big[(I_1\cdot Q_2+Q_1\cdot I_2)-0.5\\ &\quad \cdot(I_3+I_4)\cdot(Q_3+Q_4)\big]^2\Big]^{0.5}. \quad \text{(A.6)}\end{aligned}$$

Thus, the maximum polarization depends on all eight quadratures of the SM elements, i.e.,

$$m_0=f_m(I_1,Q_1,I_2,Q_2,I_3,Q_3,I_4,Q_4).$$

B. Orientation Angle

With (46) for the difference of the in-phase quadratures $I_1^{(s)}-I_2^{(s)}$, we write for the cosine of the doubled orientation angle in the eigenpolarization basis

$$\cos(2\theta_0)=\frac{\left(I_1^{(s)}-I_2^{(s)}\right)+(b+f)\cdot\sin(2\theta_0)\cdot\sin(2\varepsilon_0)}{a-d}. \quad \text{(A.7)}$$

Accordingly, from the equation for $Q_1^{(s)}-Q_2^{(s)}$ we find

$$\sin(2\theta_0)\cdot\sin(2\varepsilon_0)=\frac{\left(Q_1^{(s)}-Q_2^{(s)}\right)-(b-f)\cdot\cos(2\theta_0)}{a+d}. \quad \text{(A.8)}$$

Substitution in (A.7) gives

$$\cos(2\theta_0)=\frac{\left(I_1^{(s)}-I_2^{(s)}\right)\cdot(a+d)+\left(Q_1^{(s)}-Q_2^{(s)}\right)\cdot(b+f)}{|\dot\lambda_{01}|^2-|\dot\lambda_{02}|^2}. \quad \text{(A.9)}$$

We derive from the equation for $2I_3^{(s)}$

$$\sin(2\theta_0)=\frac{2I_3^{(s)}-(b+f)\cdot\cos(2\theta_0)\cdot\sin(2\varepsilon_0)}{a-d} \quad \text{(A.10)}$$

and from the equation for $2Q_3^{(s)}$

$$\cos(2\theta_0)\cdot\sin(2\varepsilon_0)=-\frac{2Q_3^{(s)}-(b-f)\cdot\sin(2\theta_0)}{a+d} \quad \text{(A.11)}$$

and obtain for $\sin(2\theta_0)$

$$\sin(2\theta_0)=\frac{2I_3^{(s)}\cdot(a+d)+2Q_3^{(s)}\cdot(b+f)}{|\dot\lambda_{01}|^2-|\dot\lambda_{02}|^2}. \quad \text{(A.12)}$$

With

$$\frac{b+f}{a+d}=\frac{Q_1^{(s)}+Q_2^{(s)}}{I_1^{(s)}+I_2^{(s)}} \quad \text{(A.13)}$$

which follows from the first two equations of (46), the expression for the ratio of $\sin(2\theta_0)$ (A.12) and $\cos(2\theta_0)$ (A.9) can be represented as $\tan(2\theta_0)=(2I_3^{(s)}\cdot(I_1^{(s)}+I_2^{(s)})+2Q_3^{(s)}\cdot(Q_1^{(s)}+Q_2^{(s)}))/(((I_1^{(s)})^2+(Q_1^{(s)})^2)-((I_2^{(s)})^2+(Q_2^{(s)})^2))$. This expression includes the quadrature elements $I_i^{(s)}$, $Q_i^{(s)}$ of the symmetric part of the SM directly. Using the expressions in (44), which connect these quadratures with the elements' quadratures of the initial matrix, we can rewrite the last equation into

$$\tan(2\theta_0)=\frac{(I_1+I_2)\cdot(I_3+I_4)+(Q_1+Q_2)\cdot(Q_3+Q_4)}{(I_1^2+Q_1^2)-(I_2^2+Q_2^2)}. \quad \text{(A.14)}$$

Therefore, the orientation angle of the eigenpolarization basis of an arbitrary radar object is written as

$$\theta_0=0.5\cdot\mathrm{atan}\left(\frac{(I_1+I_2)\cdot(I_3+I_4)+(Q_1+Q_2)\cdot(Q_3+Q_4)}{(I_1^2+Q_1^2)-(I_2^2+Q_2^2)}\right). \quad \text{(A.15)}$$

It is easy to see that the orientation angle also depends on all eight quadratures of the SM elements, i.e., $\theta_0=f_0(I_1,Q_1,I_2,Q_2,I_3,Q_3,I_4,Q_4)$.

C. Ellipticity Angle

From the first equation of (A.10), the expression

$$a+d=\frac{I_1^{(s)}+I_2^{(s)}}{\cos(2\varepsilon_0)} \quad \text{(A.16)}$$

is substituted into the fourth and sixth equations of (46), i.e., in the equation for the difference of the imaginary parts of the diagonal elements of the SM symmetric part and in the equation for the doubled imaginary part of the off-diagonal elements. As a result, these equations can be written as

$$(b-f)\cdot\cos(2\theta_0)+\left(I_1^{(s)}+I_2^{(s)}\right)\cdot\sin(2\theta_0)\cdot\tan(2\varepsilon_0)=Q_1^{(s)}-Q_2^{(s)} \quad \text{(A.17)}$$

$$(b-f)\cdot\sin(2\theta_0)-\left(I_1^{(s)}+I_2^{(s)}\right)\cdot\cos(2\theta_0)\cdot\tan(2\varepsilon_0)=2Q_3^{(s)}. \quad \text{(A.18)}$$

With

$$b-f=\frac{2Q_3^{(s)}+\left(I_1^{(s)}+I_2^{(s)}\right)\cdot\cos(2\theta_0)\cdot\tan(2\varepsilon_0)}{\sin(2\theta_0)} \quad \text{(A.19)}$$

and making a simple transformation, we get

$$\tan(2\varepsilon_0)=\frac{\left(Q_1^{(s)}-Q_2^{(s)}\right)\cdot\sin(2\theta_0)-2Q_3^{(s)}\cdot\cos(2\theta_0)}{I_1^{(s)}+I_2^{(s)}}. \quad \text{(A.20)}$$

Using (48), (A.20) will take its final form

$$\tan(2\varepsilon_0)=\frac{(Q_1-Q_2)\cdot\sin(2\theta_0)-(Q_3+Q_4)\cdot\cos(2\theta_0)}{I_1+I_2} \quad \text{(A.21)}$$

and the ellipticity angle of the eigenpolarization basis of an arbitrary radar object becomes

$$\varepsilon_0=0.5\cdot\mathrm{atan}\left(\frac{(Q_1-Q_2)\cdot\sin(2\theta_0)-(Q_3+Q_4)\cdot\cos(2\theta_0)}{I_1+I_2}\right). \quad \text{(A.22)}$$

This function contains six quadratures of the initial scattering matrix and the trigonometric functions of the doubled orientation angle $2\theta_0$, i.e., $\varepsilon_0=f_\varepsilon(I_1,Q_1,I_2,Q_2,Q_3,Q_4,2\theta_0)$.

D. Skip Angle

Since the arguments of the complex eigenvalues (where the absolute phase is taken into consideration) are equal to

$$\arg\{\dot{\lambda}_{01}\} = \phi_0 + 2\nu_0 \quad \arg\{\dot{\lambda}_{02}\} = \phi_0 + 2\nu_0$$

the parameter ν_0 can be found as the difference of these arguments

$$4\nu_0 = \arg\{\dot{\lambda}_{01}\} - \arg\{\dot{\lambda}_{02}\}$$

or

$$4\nu_0 = \operatorname{atan}\left(\frac{b}{a}\right) - \operatorname{atan}\left(\frac{f}{d}\right). \qquad \text{(A.23)}$$

Supposing the orientation and ellipticity angles (θ_0, ε_0) are known, we can make the following set of four equations:

$$\begin{cases} (a+d)\cdot\cos(2\varepsilon_0) = I_1^{(s)} + I_2^{(s)} \\ (b+f)\cdot\cos(2\varepsilon_0) = Q_1^{(s)} + Q_2^{(s)} \\ (a-d)\cdot\sin(2\theta_0) + (b+f)\cdot\cos(2\theta_0)\cdot\sin(2\varepsilon_0) = 2I_3^{(s)} \\ (b-f)\cdot\sin(2\theta_0) - (a+d)\cdot\cos(2\theta_0)\cdot\sin(2\varepsilon_0) = 2Q_3^{(s)}. \end{cases} \qquad \text{(A.24)}$$

To solve this equation set by the Kramer's method, we must find the determinant (see first equation at bottom of page). Determinants D_a, D_b, D_d, and D_f, can be derived from D by replacing the corresponding columns by the column consisting of absolute terms. For example, the determinant D_a has the form shown in the last equation at bottom of page.

After some transformations and simplifications, we present the determinants in the final form of

$$\begin{aligned} D &= (1+\cos(4\varepsilon_0))\cdot(1-\cos(4\theta_0)) \\ D_a &= V1 + (V2 - V3) \quad D_b = U1 - (U2 + U3) \\ D_d &= V1 - (V2 - V3) \quad D_f = U1 + (U2 + U3) \end{aligned} \qquad \text{(A.25)}$$

where, by taking (44) into consideration, we have

$$\begin{aligned} V1 &= (I_1 + I_2)\cdot\cos(2\varepsilon_0)\cdot(1-\cos(4\theta_0)) \\ V2 &= 0.5\cdot(Q_1+Q_2)\cdot\sin(4\theta_0)\cdot\sin(4\varepsilon_0) \\ V3 &= (I_3+I_4)\cdot\sin(2\theta_0)\cdot(1+\cos(4\varepsilon_0)) \\ U1 &= (Q_1+Q_2)\cdot\cos(2\varepsilon_0)\cdot(1-\cos(4\theta_0)) \\ U2 &= 0.5\cdot(I_1+I_2)\cdot\sin(4\theta_0)\cdot\sin(4\varepsilon_0) \\ U3 &= (Q_3+Q_4)\cdot\sin(2\theta_0)\cdot(1+\cos(4\varepsilon_0)). \end{aligned} \qquad \text{(A.26)}$$

If the determinant D is not equal to zero, i.e.,

$$(1+\cos(4\varepsilon_0))\cdot(1-\cos(4\theta_0)) \neq 0$$

then then equation set given in (46) has the unique solution

$$a = \frac{D_a}{D} \quad b = \frac{D_b}{D} \quad d = \frac{D_d}{D} \quad f = \frac{D_f}{D}.$$

The "preliminary" value of the skip angle can then be written as

$$\nu_0^{(p)} = 0.25\cdot\left[\operatorname{atan}\left(\frac{D_\mathrm{b}}{D_\mathrm{a}}\right) - \operatorname{atan}\left(\frac{D_\mathrm{f}}{D_\mathrm{d}}\right)\right]$$

or

$$\nu_0^{(p)} = 0.25\cdot\left[\operatorname{atan}\left(\frac{U1-(U2+U3)}{V1+(V2-V3)}\right) - \operatorname{atan}\left(\frac{U1+(U2+U3)}{V1-(V2-V3)}\right)\right]. \qquad \text{(A.27)}$$

With (49) and (50) and using some transformations, parameter ν_0 becomes

$$\nu_0 = \begin{cases} \nu_0^{(p)}, & D_a\cdot D_d > -D_b\cdot D_f \\ \nu_0^{(p)} + \frac{\pi}{4}, & D_a\cdot D_d \leq -D_b\cdot D_f, \frac{D_b}{D_a} > 0 \\ \nu_0^{(p)} - \frac{\pi}{4}, & D_a\cdot D_d \leq -D_b\cdot D_f, \frac{D_b}{D_a} < 0 \end{cases} \qquad \text{(A.28)}$$

where **au: add text to properly direct the reader** (see equation at bottom of the next page).

E. Characteristic Angle

Since the characteristic angle value is connected with the eigenvalue moduli via

$$\tan^2\gamma_0 = \frac{|\dot{\lambda}_{02}|}{|\dot{\lambda}_{01}|}$$

$$D = \begin{vmatrix} \cos(2\varepsilon_0) & 0 & \cos(2\varepsilon_0) & 0 \\ 0 & \cos(2\varepsilon_0) & 0 & \cos(2\varepsilon_0) \\ \sin(2\theta_0) & \cos(2\theta_0)\cdot\sin(2\varepsilon_0) & -\sin(2\theta_0) & \cos(2\theta_0)\cdot\sin(2\varepsilon_0) \\ -\cos(2\theta_0)\cdot\sin(2\varepsilon_0) & \sin(2\theta_0) & -\cos(2\theta_0)\cdot\sin(2\varepsilon_0) & -\sin(2\theta_0) \end{vmatrix}$$

$$D_a = \begin{vmatrix} I_1^{(s)} + I_2^{(s)} & 0 & \cos(2\varepsilon_0) & 0 \\ Q_1^{(s)} + Q_2^{(s)} & \cos(2\varepsilon_0) & 0 & \cos(2\varepsilon_0) \\ 2I_3^{(s)} & \cos(2\theta_0)\cdot\sin(2\varepsilon_0) & -\sin(2\theta_0) & \cos(2\theta_0)\cdot\sin(2\varepsilon_0) \\ 2Q_3^{(s)} & \sin(2\theta_0) & -\cos(2\theta_0)\cdot\sin(2\varepsilon_0) & -\sin(2\theta_0) \end{vmatrix}$$

and the modulus of the second complex eigenvalue can be written as

$$|\dot{\lambda}_{02}| = \left[0.5 \cdot \left(\mathrm{span}\left(\mathbf{S}^{(s)}\right) - P\right)\right]^{0.5}$$

this invariant is easily found from the equation

$$\gamma_0 = \mathrm{atan}\left(\left(\frac{\mathrm{span}\left(\mathbf{S}^{(s)}\right) - P}{\mathrm{span}\left(\mathbf{S}^{(s)}\right) + P}\right)^{0.25}\right) \tag{A.31}$$

and is, therefore, also a function of all eight quadratures of the initial scattering matrix, i.e., $\gamma_0 = f_\gamma(I_1, Q_1, I_2, Q_2, I_3, Q_3, I_4, Q_4)$.

F. Absolute Phase

The determinant of the symmetric scattering matrix

$$\det\left(\mathbf{S}^{(s)}\right) = \dot{S}_{11}^{(s)} \cdot \dot{S}_{22}^{(s)} - \left(\dot{S}_{12}^{(s)}\right)^2 = \dot{\lambda}_{01} \cdot \dot{\lambda}_{02} \tag{A.32}$$

is one of the SM invariants, i.e., it does not depend on the radar polarization basis. In this respect, not only the determinant's modulus is an invariant value, but also its argument. With

$$\dot{\lambda}_{01} = |\dot{\lambda}_{01}| \cdot e^{j(2\nu_0 + \phi_0)} \quad \dot{\lambda}_{02} = |\dot{\lambda}_{02}| \cdot e^{-j(2\nu_0 - \phi_0)}$$

it is easy to write for the determinant in the radar object's eigenbasis

$$\det\left(\mathbf{S}^{(\mathrm{s})}\right) = |\dot{\lambda}_{01}| \cdot |\dot{\lambda}_{02}| \cdot e^{j2\phi_0}. \tag{A.33}$$

Thus, the "preliminary" value of the absolute phase of the SM symmetric part can be found from

$$\phi_0^{(p)} = 0.5 \cdot \arg\left\{\det\left(\mathbf{S}^{(s)}\right)\right\} \tag{A.34}$$

(cf. [8 (see also comments by H. Mieras, pp. 1470–1471, and author's reply, pp. 1471–1473]) or

$$\phi_0^{(p)} = 0.5 \cdot \mathrm{atan}\left(\frac{\mathrm{Im}\left\{\det\left(\mathbf{S}^{(s)}\right)\right\}}{\mathrm{Re}\left\{\det\left(\mathbf{S}^{(s)}\right)\right\}}\right). \tag{A.35}$$

Since the argument of a complex number can only be found unambiguously in the angular interval (0; 2π) or ($-\pi$; $+\pi$), the $\phi_0^{(p)}$ value (as a half of this angle) is reduced to the interval (0; π) or ($-\pi/2$; $+\pi/2$). Suppose that $\phi_0^{(p)}$ is determined in the interval (0; π), expression (A.35) only gives the true estimate of the absolute phase for positive values. Therefore, to find the correct value of the absolute phase in the interval ($-\pi$; $+\pi$), we must calculate the additional angle ($\phi_0^{(p)} - \pi$) and check which value is true. Such verification may be carried out as a result of substitution of both values $\phi_0^{(p)}$ and ($\phi_0^{(p)} - \pi$) in the expression for the quadrature of the SM symmetric part, and include a comparison of the calculated value with the initial one. By choosing the in-phase quadrature $I_3^{(s)}$, we can show that the absolute phase may be found according to the rule

$$\phi_0 = \begin{cases} \phi_0^{(p)} & G = +0.5 \cdot (I_3 + I_4) \\ \phi_0^{(p)} - \pi, & G = -0.5 \cdot (I_3 + I_4)\,. \end{cases} \tag{A.36}$$

The values of $\phi_0^{(p)}$ and G in (A.36) are equal to

$$\phi_0^{(p)} = 0.5 \cdot \mathrm{atan}\left(\frac{(I_1 \cdot Q_2 + Q_1 \cdot I_2) - 0.5 \cdot (I_3 + I_4) \cdot (Q_3 + Q_4)}{(I_1 \cdot I_2 - Q_1 \cdot Q_2) - 0.25 \cdot [(I_3 + I_4)^2 - (Q_3 + Q_4)^2]}\right) \tag{A.37}$$

and

$$G = F1 \cdot \cos\phi_0^{(p)} - F2 \cdot \sin\phi_0^{(p)} \tag{A.38}$$

where

$$\begin{aligned} F1 &= 0.5 \cdot m \cdot (1 - \tan^2\gamma_0) \\ &\quad \cdot [\cos(2\nu_0) \cdot \sin(2\theta_0) + \sin(2\nu_0) \cdot \cos(2\theta_0) \cdot \sin(2\varepsilon_0)] \\ F2 &= 0.5 \cdot m \cdot (1 + \tan^2\gamma_0) \\ &\quad \cdot [\sin(2\nu_0) \cdot \sin(2\theta_0) - \cos(2\nu_0) \cdot \cos(2\theta_0) \cdot \sin(2\varepsilon_0)]\,. \end{aligned}$$

G. Nonreciprocity Angle and Absolute Phases Difference

In this section we pay attention to the nonreciprocal aspects. We summarize the analytic forms for the polarization invariants describing properties of the antisymmetric part of the scattering matrix. The polarization invariant describing the nonreciprocal properties of an arbitrary radar object is written as

$$\dot{\xi} = \zeta \cdot e^{j\eta}$$

where ζ is the nonreciprocity angle and η is the absolute phase difference of the symmetric and antisymmetric parts of the scattering matrix. These invariants are found in the form

$$\zeta = \mathrm{atan}\left(|\dot{\xi}|\right) \quad \eta = \arg\{\dot{\xi}\}. \tag{A.39}$$

$$\nu_0^{(p)} = 0.25 \cdot \mathrm{atan}\left(\frac{0.5 \cdot P2 \cdot [(I_3 + I_3) \cdot (Q_1 + Q_2) - (Q_3 + Q_4) \cdot (I_1 + I_2)]}{P4 \cdot [(I_1 + I_2)^2 + (Q_1 + Q_2)^2] + 0.5 \cdot P3 \cdot [(I_3 + I_4) \cdot (Q_1 + Q_2) - (Q_3 + Q_4) \cdot (I_1 + I_2)] - P1 \cdot [(I_1 + I_2)^2 + (Q_1 + Q_2)^2] - 0.25 \cdot P5 \cdot [(I_3 + I_4)^2 + (Q_3 + Q_4)^2]}\right) \tag{A.29}$$

$$\begin{aligned} &P1 = \sin(4\theta_0) \cdot (1 - \cos(4\theta_0)) \cdot \sin(4\varepsilon_0) \cdot \cos(2\varepsilon_0) \qquad P2 = \cdot\sin(2\theta_0) \cdot (1 - \cos(4\theta_0)) \cdot \cos(2\varepsilon_0) \cdot (1 + \cos(4\varepsilon_0)) \\ &P3 = 2 \cdot \sin(2\theta_0) \cdot \sin(4\theta_0) \cdot \sin(4\varepsilon_0) \cdot (1 + \cos(4\varepsilon_0)) \qquad P4 = \cos^2(2\varepsilon_0) \cdot (1 - \cos(4\theta_0))^2 - 0.25 \cdot \sin^2(4\theta_0) \cdot \sin^2(4\varepsilon_0) \\ &P5 = 4 \cdot \sin^2(2\theta_0) \cdot (1 + \cos(4\varepsilon_0))^2\,. \end{aligned} \tag{A.30}$$

Taking into consideration that the modulus of the nonreciprocity factor equals

$$|\dot{\xi}| = \frac{\sqrt{2}\cdot|\dot{\Delta}|}{\|\mathbf{S}\|}$$

where $\dot{\Delta} = 0.5 \cdot (\dot{S}_{21} - \dot{S}_{12})$ and $\|\mathbf{S}\| = (|\dot{S}_{11}|^2 + |\dot{S}_{12}|^2 + |\dot{S}_{21}|^2 + |\dot{S}_{22}|^2)^{0.5}$, we can write the expression for the nonreciprocity angle as

$$\zeta = \operatorname{atan}\left(\frac{\sqrt{2}}{2} \cdot \left(\frac{(I_4-I_3)^2+(Q_4-Q_3)^2}{(I_1^2+Q_1^2)+(I_2^2+Q_2^2)+(I_3^2+Q_3^2)+(I_4^2+Q_4^2)}\right)^{0.5}\right) \quad \text{(A.40)}$$

and the difference in absolute phases of the SM symmetric and antisymmetric parts as

$$\eta = \arg\{\dot{\Delta}\} = \operatorname{atan}\left(\frac{Q_4 - Q_3}{I_4 - I_3}\right). \quad \text{(A.41)}$$

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